MTH 127 Student Learning Outcomes (SLO) Assessment  
SLOAT Fall 2010 Final Report

Introduction

MTH 127 is a basic calculus course designed for Business majors who plan on transferring to colleges or universities that require calculus for their Business Bachelor’s Degrees. MTH 127 is a very intense course, as it numerous covers topics ranging from pre-calculus concepts to using simple ordinary differential equations to mathematically model various business-type applications. The MTH 127 course outline lists the course goals as follows:

1. demonstrate knowledge of the fundamental concepts and theories from pre-calculus, calculus, and introductory ordinary-differential equations;
2. utilize various pre-calculus, calculus, and introductory differential equation problem-solving and critical-thinking techniques to set up and solve applied problems in finance, economics, geometry, sciences, and other fields;
3. communicate accurate mathematical terminology and notation in written and/or oral form in order to explain strategies to solve problems as well as to interpret found solutions; and
4. use graphing calculators effectively as a tool to solve such problems as those described above.

SLOAT Assessment Plan

The SLOAT Fall 2010 MTH 127 Student Learning Outcomes (SLO) assessment, which was conducted by Susan Gaulden, focused on course goals 1 and 2 only. Data was collected on the performance of all students attending MTH 127 section 001 (Susan Gaulden’s section), which originally included 37 students but was ended with only 25 students at the conclusion of the semester. Although this is a small sample size, it is important to note that only 2 sections of MTH 127 were offered in Fall 2010 and only a total of 69 students were enrolled at the start of the semester. Thus, the initial sample size was 37/69 or approximately 54% of the entire MTH 127 student population.

Almost all questions from 6 tests given in class throughout the semester and from the cumulative final exam in MTH 127 section 001 were blueprinted to the specific measurable course performance objectives (MPOs) that correspond to course goals 1 and 2. These tests and the final exam – showing the question-to-MPO blueprint as well as collected SLO data – are included as Appendix A in this report. Also, the day before each test was given in MTH 127 section 001, all students present were asked to complete brief surveys to self-assess their abilities to accomplish the MPOs that were included on the next day’s test. In addition to students reporting their level of confidence with specific skills on the surveys, they also were
asked to identify barriers that exist in their lives and prevent them from Excelling at math, to list practices that would help them do better at math, to indicate things that the instructor could do differently to improve the class, and address other similar concerns. These surveys, including the results of the surveys, are included as Appendix B in this report.

Assessment Results/Findings

Survey (Student Perception) Results – As mentioned above, the results of the surveys are given in their entirety in Appendix B of this report, but interesting results from the student surveys include the following:

53% of respondents indicated that they have test anxiety and 36% don’t study enough because of overcommitted schedules. Both of these items were reported as barriers that keep students from Excelling in math.

36% of respondents offered that they should do more practice and an additional 25% said they should study more to improve their chances of success in math. This means that 61% of the students in MTH 127 felt that they needed to work harder on their own to achieve better results in the course.

Although 42% of respondents asked the instructor to slow down and to not assume they know the material from MTH 100 even though that is the course prerequisite, 25% of respondents felt that the instructor needed to improve ‘nothing’ at all to help students do better at math.

Regarding the survey components, which required students to self-assess their abilities to accomplish the MPOs that were included on the next day’s test, the following noteworthy results were reported: (Note: The percentages given in parentheses below indicate the number of students who agreed with the statement ‘I know how to do this’ for the given skill/math topic on the survey.)

Students were very confident in their self-assessed abilities to perform the following skills: evaluate polynomial functions (MPO 1.1, 97%), calculate derivatives using the product rule (MPO 1.5, 93%), solve quadratic equations (MPO 1.2, 83%), and calculate derivatives using the quotient rule (MPO 1.5, 81%).

Students were least confident in their self-assessed abilities to perform the following skills: solve related rates problems (MPO 2.3, 15%), determine a specified volume of revolution (MPO 1.8, 23%), calculate derivatives using logarithmic differentiation (MPO 1.5, 26%), and calculate derivatives of logarithmic functions (MPO 1.5, 27%).

Student self-assessment seemed to significantly overestimate their abilities to perform the following skills: evaluate polynomial functions (MPO 1.1, 97% perceived they knew how to
do this versus only 47% who actually knew how to do this on Test 1), graph a function – indicate the relative extrema, inflection points, concavity, and y-intercept (MPO 1.1, 63% perceived they knew how to do this versus only 23% who actually knew how to do this on Test 3), solve logarithmic equations (MPO 1.2, 70% perceived they knew how to do this versus only 36% who actually knew how to do this on Test 4), and solve exponential equations (MPO 1.2, 78% perceived they knew how to do this versus only 54% who actually knew how to do this on Test 4).

Student self-assessment seemed to significantly underestimate their abilities to perform the following skills: solve elasticity of demand problems (MPO 2.6, only 33% perceived they knew how to do this versus 91% who actually knew how to do this on Test 5), integrate functions (MPO 1.7, only 41% perceived they knew how to do this versus 96% who actually knew how to do this on Test 5), calculate definite integrals by applying the Fundamental Theorem of Calculus (MPO 1.7, only 45% perceived they knew how to do this versus 91% who actually knew how to do this on Test 5), and solve growth and decay problems (MPO 2.5, only 55% perceived they knew how to do this versus 100% who actually knew how to do this on Test 5).

‘Indirect’ Factors that Affect Student Learning – Anecdotally, most instructors describe attendance as having a significant impact on student performance in classes. As was mentioned above (see Assessment Plan section), the initial enrollment of MTH 127 Section 001 included 37 students, but only 25 students completed the class. This means that 12 students (32%) withdrew from (did not complete) the class. Attendance data for these 12 non-completers, along with those who completed (did not withdraw from) MTH 127 Section 001 students, are given in the table below. **NOTE:** For students who withdrew from the class, the % of classes missed is calculated by dividing the # of absences up to the given test date by the number of classes held up to that point in the semester. On average, students who withdrew missed 27.8% of the classes held up to that point in the semester.

---

<table>
<thead>
<tr>
<th>Final Course Grade</th>
<th># of Students</th>
<th># of Classes Missed</th>
<th>Average # (%) of Classes Missed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>0, 1, 1, 1, 2, 3, 4, 5, 5, 10</td>
<td>3.2 (7.6%)</td>
</tr>
<tr>
<td>B+</td>
<td>2</td>
<td>0, 1 → 0.5</td>
<td>1.2%</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>1, 2, 3 → 2.0</td>
<td>4.8%</td>
</tr>
<tr>
<td>C+</td>
<td>4</td>
<td>1, 6, 6, 9 → 5.5</td>
<td>13.1%</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>not applicable</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>4</td>
<td>1, 1, 11, 21 → 8.5</td>
<td>20.2%</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>26, 30 → 28</td>
<td>66.7%</td>
</tr>
<tr>
<td>W</td>
<td>12</td>
<td>1 withdrew before Test 1: 6 → 6 (75.0%)</td>
<td>4 withdrew before Test 2: 0, 1, 2, 4 → 1.8 (11.7%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1 withdrew before Test 3: 4 → 4 (18.2%)</td>
<td>2 withdrew before Test 4: 2, 6 → 4 (14.3%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 withdrew immediately after Test 4: 2, 5, 8, 8 → 5.8 (19.8%)</td>
<td></td>
</tr>
</tbody>
</table>
Data from the above table is presented in the bar graph below. This data (given in the table above or the graph below) seems to support the hypothesis that students who miss more class sessions do not perform as well in the class. In fact, students who passed the class were absent from 7.6% of the sessions on average in contrast to the 25.3% of sessions on average missed by students who did not pass the class. This finding should be shared with students who enroll in MTH 127 early in the semester so they are warned that attending class can significantly increase their chance of succeeding in the course.

The Effect of Missing Classes on Final Course Grade in MTH 127

<table>
<thead>
<tr>
<th>% of Classes Missed</th>
</tr>
</thead>
<tbody>
<tr>
<td>66.7%</td>
</tr>
<tr>
<td>27.8%</td>
</tr>
<tr>
<td>20.2%</td>
</tr>
<tr>
<td>13.1%</td>
</tr>
<tr>
<td>4.8%</td>
</tr>
<tr>
<td>1.2%</td>
</tr>
<tr>
<td>7.6%</td>
</tr>
</tbody>
</table>

Blueprinted Tests/Exam (Student Performance) Results – On every test and the final exam, all questions were blueprinted to the MPOs related to course goals 1 or 2 that were covered on that test/exam. (See Appendix A for blueprinted tests and final exam as well as collected data.) Each student, based on his/her answers on each of the blue-printed questions, was classified by Susan Gaulden, the instructor/grader, as falling into one of the three following categories: knows how to do this, has some idea how to do this but needs more practice, or does not know how to do this. Note that often there was more than one question on a given test/exam that blueprinted to a specific MPO. For example, there were 2 questions on Test 4 involving solving exponential equations. In this case, a student who got both questions correct was counted as ‘knowing how to do this’, a student who got one of the questions correct or both of the questions partly correct was counted as ‘having some idea how to do this but needing more practice’, and a student who got both questions wrong was counted as ‘not knowing how to do this.’ The number of students in each category was tallied for each test and for the final exam.

Data was collected for the entire class to determine which course objectives were achieved by the most students and which were mastered by the fewest students. Since questions related to the same MPO often appeared on more than one test or the final exam, it is important to clarify how the percentages of students who ‘know how to do this’, who ‘have some idea how to do this’, and who ‘do not know how to do this’ were calculated for each MPO. An example of such calculations is given below.
MPO 1.3 actually involves the following 2 distinct skills: determining limits and determining continuity and/or differentiability of a function at a specified value. Limit questions were present on Test 2 and continuity/differentiability questions were asked on both Test 2 and on the final exam. On Test 2, 25 students got the limit questions correct, 6 got them somewhat correct, and 0 got them incorrect. On Test 2, 18 students got the continuity/differentiability questions correct, 8 got the somewhat correct, and 5 got them incorrect. On the final exam, 7 students got the continuity/differentiability questions correct, 5 got the somewhat correct, and 8 got them incorrect. This means that overall, the percentages of students in each classification (‘know how to do this’, ‘have some idea how to do this’, and ‘do not know how to do this’) – where this means MPO 1.3 – are calculated as follows: 
\[
\frac{25+18+7}{25+6+0} + \frac{18+8+5}{(18+8+5)} + \frac{7+5+8}{(7+5+8)} = \frac{61}{68+8+5} \approx 23\% \ 
\text{have some idea how to do MPO 1.3, and}
\frac{6+8+5}{(25+6+0)+(18+8+5)+(7+5+8)} \approx \frac{16}{23} \% \ 
\text{do not know how to do MPO 1.3.}
\]

In this SLOAT study, an MPO is considered to be achieved if at least 70% of the students were counted as ‘knowing how to do this’ on blueprinted questions from all relevant tests/exams. An MPO is partially achieved if between 50% and 70% of the students were counted as ‘knowing how to do this’ on blueprinted questions from all relevant tests/exams. And an MPO is not achieved if less than half of the students were counted as ‘knowing how to do this’ on blueprinted questions from all relevant tests/exams. Although the entire data, which includes the percentages of each classification of students on each test and on the final exam, is included in Appendix B, the results of the analysis grouped by course goal and organized by MPO are as follows:

- **Course Goal 1**: demonstrate knowledge of the fundamental concepts and theories from pre-calculus, calculus, and introductory ordinary-differential equations

MPO 1.1 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.1, evaluate and graph (using the first-derivative and second-derivative tests as appropriate) polynomial, piecewise, composite, exponential, logarithmic, and multi-variable functions, were classified as follows: 52% know how to do this, 32% have some idea how to do this, and 15% do not know how to do this. It can be concluded, then, that students only partially achieved this MPO. It is important to note the following specifics: students performed best on evaluating multi-variable functions (100% correctly answered this question on the final exam); many students did well on graphing polynomial functions (68% correctly answered this question on Test 2) and evaluating piecewise functions (64% correctly answered this question on Test 1); students had difficulty mastering graphing piecewise functions (34% incorrectly answered this question on Test 1).
MPO 1.2 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.2, solve linear, quadratic, exponential, and logarithmic equations, were classified as follows: 70% know how to do this, 21% have some idea how to do this, and 12% do not know how to do this. It can be concluded, then, that students achieved this MPO. It is important to note the following specifics: 100% correctly solved exponential equations, 75% correctly solved quadratic equations, and 70% correctly solved logarithmic equations on the final exam.

MPO 1.3 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.3, determine limits, continuity, and differentiability of given functions at specified values, were classified as follows: 61% know how to do this, 23% have some idea how to do this, and 16% do not know how to do this. It can be concluded, then, that students partially achieved this MPO. It is important to note the following specifics: students performed best on determining limits (81% correctly answered these questions on Test 2) and worst on determining continuity and/or differentiability of a function at a specified value (40% incorrectly answered this question on the final exam).

MPO 1.4 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.4, determine a derivative of a function by using limits and difference quotients, were classified as follows: 38% know how to do this, 13% have some idea how to do this, and 50% do not know how to do this. It can be concluded, then, that students did not achieve this MPO. It is important to note the following specifics: although 52% of the students correctly answered this question on Test 1, all of the students who answered a related extra credit question on the final exam answered it incorrectly. This means that students certainly seem to forget how to calculate derivatives by using the definition of the derivative once they have learned some of the other (shortcut) rules of differentiation such as the power rule, the product rule, the chain rule, etc.

MPO 1.5 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.5, calculate first, second, or partial derivatives of polynomial, rational, exponential, and logarithmic functions by using rules of differentiation including the product, quotient, and chain rules and implicit and logarithmic differentiation, were classified as follows: 55% know how to do this, 28% have some idea how to do this, and 17% do not know how to do this. It can be concluded, then, that students partially achieved this MPO. It is important to note the following specifics: students performed best on calculating derivatives using the product, quotient, and chain rules (80% correctly answered questions requiring differentiation via the product and quotient rules on Test 3 and 79% correctly answered questions requiring the chain rule on Test 4) and on calculating the derivatives of exponential functions (68% correctly answered these questions on Test 4); students had difficulty calculating derivatives of logarithmic functions (32% incorrectly answered this question on Test 4) and calculating derivatives using logarithmic differentiation (32% incorrectly answered this question on Test 4).

MPO 1.6 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.6, calculate Riemann sums to estimate definite integrals, were classified as follows: 83% know how to do this, 17% have some idea how to do this, and 0% do not know how to do this. It can be concluded, then, that students achieved this MPO.

MPO 1.7 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.7, apply the Fundamental Theorem of Calculus to calculate integrals of single variable functions and determine the areas between given curves, were classified as follows: 60% know how to do this, 29% have some idea how to do this, and 10% do not know how to do this. It can be concluded, then, that students partially achieved this MPO. It is important to note the following specifics: students performed best on calculating definite integrals by applying the Fundamental Theorem of Calculus (74% correctly answered these questions on Test 5 and the final exam); all students at least had some idea how to determine the area between curves (35% answered this question correctly and 65% answered this question somewhat correctly on Test 6); and although 96% of the students got the integrating function questions correct on Test 5, 35% incorrectly answered this question on the final exam.

MPO 1.8 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 1.8, determine a specified volume of revolution, were classified as follows: 63% know how to do this, 25% have some idea how to do this, and 13% do not know how to do this. It can be concluded, then, that students partially achieved this MPO.

The MPO analysis for course goal 1 given in detail above is summarized in the chart on the next page. It is notable that for course goal 1, 2 (25%) of the MPOs were achieved, 5 (62.5%) of the MPOs were partially achieved, and 1 (12.5%) of the MPOs was not achieved.
Course Goal 1: Demonstrate knowledge of the fundamental concepts and theories from pre-calculus, calculus, and introductory ordinary-differential equations.

<table>
<thead>
<tr>
<th>Course Goal</th>
<th>MPO</th>
<th>achieved</th>
<th>partially achieved</th>
<th>not achieved</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 evaluate and graph (using the first-derivative and second-derivative tests as appropriate) polynomial, piecewise, composite, exponential, logarithmic, and multi-variable functions</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>1.2 solve linear, quadratic, exponential, and logarithmic equations</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.3 determine limits, continuity, and differentiability of given functions at specified values</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4 determine a derivative of a function by using limits and difference quotients</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>1.5 calculate first, second, or partial derivatives of polynomial, rational, exponential, and logarithmic functions by using rules of differentiation including the product, quotient, and chain rules and implicit and logarithmic differentiation</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>1.6 calculate Riemann sums to estimate definite integrals</td>
<td>✓</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.7 apply the Fundamental Theorem of Calculus to calculate integrals of single variable functions and determine the areas between given curves</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8 determine a specified volume of revolution</td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Course Goal 2: utilize various pre-calculus, calculus, and introductory differential equation problem-solving and critical-thinking techniques to set up and solve applied problems in finance, economics, geometry, sciences, and other fields

MPO 2.1 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 2.1, solve compound interest, present value, and future value problems, were classified as follows: 68% know how to do this, 21% have some idea how to do this, and 11% do not know how to do this. It can be concluded, then, that students partially achieved this MPO. It is important to note the following specifics: students performed best on solving present value and compound interest problems (91% correctly answered present-value problems and 87% correctly answered compound-interest problems on Test 5); students had difficulty solving continuous-stream-future-value problems (25% incorrectly answered this question on Test 6).

MPO 2.2 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 2.2, solve marginal cost, marginal profit, and marginal
revenue problems by using differentiation and integration as necessary, were classified as follows: 70% know how to do this, 30% have some idea how to do this, and 0% do not know how to do this. It can be concluded, then, that students achieved this MPO.

MPO 2.3 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 2.3, solve rate-of-change and related rates problems, were classified as follows: 30% know how to do this, 32% have some idea how to do this, and 38% do not know how to do this. It can be concluded, then, that students did not achieve this MPO. It is important to note the following specifics: students performed much better on rate-of-change problems (52% correctly answered this problem on Test 2) than on related-rates problems (55% incorrectly answered this problem on the final exam).

MPO 2.4 → overall, students answering the questions on any given test/exam throughout the semester that addressed MPO 2.4, solve optimization problems (in geometry, finance, inventory control, etc.) including those involving functions of several variables, were classified as follows: 60% know how to do this, 20% have some idea how to do this, and 20% do not know how to do this. It can be concluded, then, that students partially achieved this MPO. It is important to note the following specifics: 67% of students answered the economic order quantity (EOQ), i.e., inventory control problem, correctly and an additional 13% answered this problem somewhat correctly on Test 3.

MPO 2.5 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 2.5, solve growth and decay problems (in finance, biology, chemistry, physics, etc.), were classified as follows: 100% know how to do this, 0% have some idea how to do this, and 0% do not know how to do this. It can be concluded, then, that students achieved this MPO.

MPO 2.6 → overall, students answering questions on any given test/exam throughout the semester that addressed MPO 2.6, solve elasticity of demand problems, as follows: 72% know how to do this, 12% have some idea how to do this, and 16% do not know how to do this. It can be concluded, then, that students achieved this MPO.

The MPO analysis for course goal 2 given in detail above is summarized in the chart on the next page. It is notable for course goal 2, 3 (50%) of the MPOs were achieved, 2 (33.3%) of the MPOs were partially achieved, and 1 (16.7%) of the MPOs was not achieved.
The bar graph below presents the level of student achievement in MTH 127 of the various MPOs for course goals 1 and 2. As can be seen in the bar graph, when course goals 1 and 2 are examined collectively, 5 (36%) of the MPOs were achieved, 7 (50%) of the MPOs were partially achieved, and 2 (14%) of the MPOs were not achieved.

**Interpretation of Results/Findings**

Many MTH 127 students self-reported that they need to spend more time studying and practicing the content covered in the course. More than half of the students in the cohort also indicated that they have test anxiety, which may keep them from excelling at math. To address
these issues, it is recommended that some class time be dedicated to and/or extra credit assignments be made that help students learn appropriate study skills and how to alleviate some of their test anxiety. Ideally, this information would be shared with students early on in the semester – even before the first test.

Students also need to be warned that some of the topics they perceive to be ‘easy’ may not be as easy as they think. When this occurs students sometimes do not spend time on these topics since they assume they will easily get these problems correct on the tests. To address this situation, students should especially be encouraged to spend time practicing topics such as evaluating polynomial functions, graphing functions (by using the First and Second Derivative Tests), and solving exponential and logarithmic equations, as these are the skills that students most overestimated their abilities to perform. It is suggested that questions on these topics, as well as the topic students perceived as the most difficult (e.g., solving related rates problems, determining volumes of revolution, and calculating derivatives of logarithmic functions and using logarithmic differentiation), be given as in-class group problem sets prior to the relevant tests.

In this SLOAT study, students achieved the 5 MPOs listed below. It is recommended that the instructional techniques pertaining to these skills not be changed, as it seems that they are working reasonably well.

- **MPO 1.2** solve linear, quadratic, exponential, and logarithmic equations
- **MPO 1.6** calculate Riemann sums to estimate definite integrals
- **MPO 2.2** solve marginal cost, marginal profit, and marginal revenue problems by using differentiation and integration as necessary
- **MPO 2.5** solve growth and decay problems (in finance, biology, chemistry, physics, etc.)
- **MPO 2.6** solve elasticity of demand problems

Furthermore, students partially achieved the 7 MPOs listed below. Specific recommendations to improve student mastery for each skill/topic are given as necessary.

- **MPO 1.1** evaluate and graph (using the first-derivative and second-derivative tests as appropriate) polynomial, piecewise, composite, exponential, logarithmic, and multi-variable functions → **Recommendation**: Teach how to graph piecewise functions in the same class as evaluating piecewise functions is taught – use index cards to block off intervals of the x-axis that are not satisfied by each piece’s condition; include questions on graphing piecewise functions on an in-class small group assignment, the MPO 1.1 skill that students had the most difficulty mastering.
MPO 1.3  determine limits, continuity, and differentiability of given functions at specified
values ➔ **Recommendation:** Spend some more class time on determining
continuity and differentiability of functions at specified values – ask students to
evaluate functions and slopes of tangent lines at points of discontinuity to have
them see graphically that these function values do not exist.

MPO 1.5  calculate first, second, or partial derivatives of polynomial, rational, exponential,
and logarithmic functions by using rules of differentiation including the product,
quotient, and chain rules and implicit and logarithmic differentiation ➔
**Recommendation:** Include questions on calculating derivatives of logarithmic
functions and using logarithmic differentiation on an in-class small group
assignment, the MPO 1.5 skills that students had the most difficulty mastering.

MPO 1.7  apply the Fundamental Theorem of Calculus to calculate integrals of single
variable functions and determine the areas between given curves ➔
**Recommendation:** Have students identify (via a check-off list) which problems
require differentiation and which require integration while reviewing for the final
exam since some students incorrectly took derivatives instead of integrating on
the final exam integration questions.

MPO 1.8  determine a specified volume of revolution ➔ **Recommendation:** Include
questions on determining volumes of revolution on an in-class small group
assignment.

MPO 2.1  solve compound interest, present value, and future value problems ➔
**Recommendation:** Have students distinguish between future value and
continuous-stream-future-value problems; ensure that they know which formula
is applicable in which instance.

MPO 2.4  solve optimization problems (in geometry, finance, inventory control, etc.)
including those involving functions of several variables ➔ **Recommendation:**
Include optimization problems on an in-class small group assignment; also be
sure to give students choices of optimization problems on tests/the final exam.

Lastly, students did not achieve the 2 MPOs listed below. Specific recommendations to
improve student mastery for each skill/topic are given as necessary.

MPO 1.4  determine a derivative of a function by using limits and difference quotients ➔
**Recommendation:** Discuss with MAP colleagues who teach this course if this
concept, which is certainly more theoretical than practical, should be included as
a MTH 127 MPO. The difficulty with this topic is that once students learn the
‘shortcut’ rules for taking derivatives, they no longer rely on the more
cumbersome method of applying the definition. If this is kept as an MPO, it is
suggested that students be given a take-home or open book in-class assignment,
which can be evaluated for student mastery of this performance objective, rather than collecting data from such questions on tests/the final exam.

MPO 2.3 solve rate-of-change and related rates problems → Recommendation: Spend more time covering related rate problems in class; distribute a supplemental handout with many more example problems on this topic as it is sparsely covered in the textbook; have students work through these problems by explicitly showing each of the following steps for every related rates problem they attempt: 1) draw a picture (if necessary) illustrating the problem; 2) identify variables and constants given or asked for in the problem; 3) identify rates given or asked for in the problem; 4) write an algebraic equation relating all variables and constants given in the problem; 5) implicitly differentiation the equation with respect to time; 6) substitute known values for the variables and rates; and 7) solve for the unknown variable or rate.

It is recommended that data be collected and analyzed that pertain to the specific measurable course performance objectives (MPOs) of course goals 3 and 4 to determine the level of student achievement on these mathematical topics/skills.

Summary

Students reported needing to spend more time practicing and studying for MTH 127, as well as some guidance to deal with test anxiety. In this SLOAT study, student achievement of two of the four course goals was determined. These two goals are as follows: 1) demonstrate knowledge of the fundamental concepts and theories from pre-calculus, calculus, and introductory ordinary-differential equations; and 2) utilize various pre-calculus, calculus, and introductory differential equation problem-solving and critical-thinking techniques to set up and solve applied problems in finance, economics, geometry, sciences, and other fields. Students achieved 2, partially achieved 5, and did not achieve 1 of the 8 measurable performance objectives related to course goal 1. Furthermore, students achieved 3, partially achieved 2, and did not achieve 1 of the 6 measurable performance objectives related to course goal 2. This means that students are certainly learning some required content/skills in MTH 127, but they are not necessarily mastering it to a desired level.

Recommendations for student learning and teaching improvement consist of the following:

• adopt specific teaching strategies for certain skills
• spend more class time on identified content as well as guiding students to enhance study skills and address test anxiety
• distribute necessary supplemental handouts with more examples for certain topics (e.g., related rates problems)
• require students to write out the steps to solve certain application problems (e.g., related rates problems)

• have students complete in-class small group assignments that include questions on specific topics

• ask students to (on sight) determine/distinguish between mathematical skills necessary to solve various problems

• give students choices of similar application problems on tests so they are able to exert their preference and still apply the same math to show their abilities

• collect some SLO assessment data from instruments other than tests/exams (e.g., in-class work, selected homework, etc.)

NOTE: Course-embedded SLO assessment is most valuable when findings and recommendations are shared with and solicited from colleagues who are also familiar with the course. For this reason, the data collected and analyzed in the MTH 127 SLOAT study as well as recommendations made to improve student learning and teaching will be shared with other mathematics instructors at a Division meeting in early Spring 2011. Some elements of the SLOAT experience, the MTH 127 SLOAT assessment plan, and the actual SLO findings will be reported to College faculty at the ECC SLO Assessment Symposium in February 2011.
Appendix A
1) Compute $f(1)$ and $f(2)$ given $f(x) = \begin{cases} \pi x^2 & \text{for } x < 2 \\ 1 + x & \text{for } 2 \leq x < 2.5 \\ 4x & \text{for } 2.5 < x \end{cases}$ (8 points)

2) Describe the domain of $\frac{8x}{(x-1)(x-2)}$. (8 points)

3) Given $f(x) = x^5$ and $g(x) = \frac{x}{1-x}$, calculate $f$ composed with $g$; that is determine $f(g(x))$. (8 points)

4) If $g(t) = 4t - t^2$, find $\frac{g(t+h) - g(t)}{h}$ and simplify. (8 points)

5) Find the equation of the line that is perpendicular to $y + x = 0$ and contains the point $(2, 0)$. (8 points)

6) Find the points of intersection of the curves $y = 2x^2 - 5x - 6$ and $y = 3x + 4$. (10 points)

7) Find the point on the graph of $y = x^2$ where the tangent line is parallel to the line $2x + 3y = 4$. (10 points)

8) In some cities you can rent a car for $18 per day and $.20 per mile. If the car is to be rented for one day, express the total rental expense as a function of the number $x$ of miles driven. (10 points)

9) Assume that a couple invests $1000 upon the birth of their daughter. Assume that the investment earns 6.8% compounded annually. What will the investment be worth on the daughter’s 18th birthday? (HINT: $A = P \left(1 + \frac{r}{n}\right)^{nt}$ where $A$ is the amount in the account, $P$ is the principal invested, $r$ is the annual interest rate, $n$ is the number of interest periods per year, and $t$ is the number of years.) (10 points)
10) **CHOOSE** and complete **ONE** of the two problems below. You must you show your work to receive maximum credit. **PLEASE NOTE:** If you complete more than one part, only the first one shown will be graded unless otherwise indicated. (10 points)

a) An average sale at a small florist shop is $21, so the shop’s weekly revenue function is
\[ R(x) = 21x, \] where \( x \) is the number of sales in 1 week. The corresponding weekly cost is
\[ C(x) = 9x + 800 \] dollars. If the profit is $1000 for 1 week, what is the revenue for 1 week?

OR

b) A frozen yogurt stand makes a profit of \( P(x) = .40x - 80 \) dollars when selling \( x \) scoops of yogurt per day. How many more scoops of yogurt will have to be sold to raise the daily profits from $30 to $40?

11) **CHOOSE** and complete **ONE** of the two problems below. You must you show your work to receive maximum credit. **PLEASE NOTE:** If you complete more than one part, only the first one shown will be graded unless otherwise indicated. (10 points)

a) After \( t \) hours of operation, an assembly line has assembled \( A(t) = 20t - \frac{1}{2}t^2 \) power lawn mowers, \( 0 \leq t \leq 10 \). Suppose that the factory’s cost of manufacturing \( x \) units is \( C(x) \) dollars, where \( C(x) = 3000 + 80x \). What is the cost of the first 2 hours of operation?

OR

b) When the owner of a gas station sets the price of 1 gallon of unleaded gasoline at $4.10, he can sell approximately 1500 gallons per day. When he sets the price at $4.25 per gallon, he can sell approximately 1250 gallons per day. Let \( G(x) \) denote the number of gallons of unleaded gasoline sold per day when the price is \( x \) dollars. Assume that \( G(x) \) is a linear function of \( x \). Approximately how many gallons will be sold per day if the price is set at $4.35 per gallon?

Extra Credit:

1) Sketch the graph of the function \( f(x) = \begin{cases} 
3 & \text{for } x < 2 \\
2x + 1 & \text{for } x \geq 2 
\end{cases} \) (2 points)

2) Is the point \( \left( \frac{1}{2}, \frac{2}{5} \right) \) on the graph of the function \( g(x) = \frac{3x - 1}{x^2 + 1} \)? (2 points)

3) An office supply firm finds that the number of fax machines sold in year \( x \) is given approximately by the function \( f(x) = 50 + 4x + \frac{1}{2}x^2 \), where \( x = 0 \) corresponds to 1990. Find the number of fax machines sold in 1992. (2 points)
### SLO Assessment Results for Test 1:

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the test</th>
<th>Students who ‘knew how to do this’</th>
<th>Students who ‘had some idea how to do this but need more practice’</th>
<th>Students who ‘did not know how to do this’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluate polynomial functions (MPO 1.1): questions 3, 4, 10, 11, EC2 &amp; EC3</td>
<td>17 = 47%</td>
<td>15 = 42%</td>
<td>4 = 11%</td>
</tr>
<tr>
<td>Evaluate piecewise functions (MPO 1.1): question 1</td>
<td>23 = 64%</td>
<td>5 = 14%</td>
<td>8 = 22%</td>
</tr>
<tr>
<td>Graph piecewise functions (MPO 1.1): question EC1</td>
<td>10 = 34%</td>
<td>9 = 31%</td>
<td>10 = 34%</td>
</tr>
<tr>
<td>Solve quadratic functions (MPO 1.2): question 6</td>
<td>29 = 81%</td>
<td>5 = 14%</td>
<td>1 = 3%</td>
</tr>
<tr>
<td>Solve compound interest problems (MPO 2.1): question 9</td>
<td>24 = 67%</td>
<td>9 = 25%</td>
<td>3 = 8%</td>
</tr>
</tbody>
</table>

**NOTE:** Questions 2, 5, 7 & 8 were not blueprinted on this test.
1) Determine each limit below if it exists. If either limit does not exist, be sure to state so and indicate why. (4 points each)
   a) \( \lim_{x \to 4} (1 - 6x) \)
   b) \( \lim_{x \to 0} \frac{x^2 + 3x}{x} \)

2) Differentiate each of the expressions below with respect to \( x \). (8 points each)
   a) \( y = x^7 + 3x + 7 - \frac{2}{x} \)
   b) \( y = (2x + 4)^3 \)

3) Calculate the second derivative of \( f(x) = (3x + 1)^3 \). (8 points)

4) Find the slope of the tangent line to the curve \( y = x^3 + 3x - 8 \) at \( (2, 6) \). (8 points)

5) Let \( P(x) \) be the profit (in dollars) from manufacturing and selling \( x \) luxury cars. Interpret \( P(100) = 90,000 \) and \( P'(100) = 1200 \). (NOTE: This means you should write one or two sentences that gives the meaning of each equation.) (8 points)

6) Determine whether the piecewise function \( f(x) = \begin{cases} 2x - 1 & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } 1 < x \end{cases} \) is continuous and/or differentiable at \( x = 1 \). Be sure to show all work to justify your answers. (10 points)

7) An analysis of the daily output of a factory assembly line shows that about \( 60t + t^2 - \frac{1}{12}t^3 \) units are produced after \( t \) hours of work, \( 0 \leq t \leq 8 \). What is the rate of production (in units per hour) when \( t = 2 \) ? (10 points)

[A-v]
8) An object moving in a straight line travels \( s(t) \) kilometers in \( t \) hours, where \( s(t) = 2t^2 + 4t \). What is the object’s velocity when \( t = 6 \), and what is the object’s acceleration when \( t = 6 \)? (10 points)

9) Refer to the graph below to answer the following questions: (2 points each)

   a. At which labeled points is the function increasing?
   b. At which labeled points is the graph concave up?
   c. Which labeled point has the most positive slope?

![Graph](image)

10) Sketch a graph of each function with properties described below. (8 points each)

   a. \( f(2) = 1; \quad f'(2) = 0; \quad \text{concave up for all } x \)
   b. \( (0, 6), (2, 3) \) and \( (4, 0) \) are on the graph; \( f''(0) = 0 \) and \( f'(4) = 0; \quad f''(x) < 0 \) for \( x < 2 \), \( f''(2) = 0 \), and \( f''(x) > 0 \) for \( x > 2 \)

Extra Credit: (2 points each)

1) Let \( f(x) = \frac{x^2 - 7x + 10}{x - 5}, \quad x \neq 5 \). If possible, define \( f(x) \) at \( x = 5 \) in a way that makes \( f(x) \) continuous for all \( x \).

2) Let \( S(x) \) represent the total sales (in thousands of dollars) for the month \( x \) in the year 2005 at a certain department store. Represent “at the end of March, the sales for this month dropped to $80,000 and were falling by about $200 a day” by equations involving \( S \) and \( S' \).

3) Use the definition of a derivative – that is, \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) – to compute the derivative of \( f(x) = x^3 \).
## SLO Assessment Results for Test 2:

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the test</th>
<th>Students who ‘knew how to do this’</th>
<th>Students who ‘had some idea how to do this but need more practice’</th>
<th>Students who ‘did not know how to do this’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph a function with given properties where information is given about its first and second derivatives (MPO 1.1): Questions 9 &amp; 10</td>
<td>21 = 68%</td>
<td>8 = 26%</td>
<td>2 = 6%</td>
</tr>
<tr>
<td>Determine limits (MPO 1.3): question 1</td>
<td>25 = 81%</td>
<td>6 = 19%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Determine whether a function is continuous and/or differentiable at a specified value for x (MPO 1.3): question 6 &amp; EC1</td>
<td>18 = 58%</td>
<td>8 = 26%</td>
<td>5 = 16%</td>
</tr>
<tr>
<td>Use the definition of a derivative to compute a derivative (MPO 1.4): question EC3</td>
<td>12 = 52%</td>
<td>4 = 17%</td>
<td>7 = 30%</td>
</tr>
<tr>
<td>Determine first and second derivatives (MPO 1.5): questions 2, 3 &amp; 4</td>
<td>13 = 42%</td>
<td>17 = 55%</td>
<td>1 = 3%</td>
</tr>
<tr>
<td>Solve rate-of-change problems (MPO 2.3): questions 5, 7, 8 &amp; EC2</td>
<td>16 = 52%</td>
<td>11 = 35%</td>
<td>4 = 13%</td>
</tr>
</tbody>
</table>
1) Find the derivative \( \frac{dy}{dx} \) of each of the following: (10 points each)

a) \( y = (x - 2)^5(x + 1)^2 \)

b) \( y = \frac{x - 1}{x + 1} \)

c) \( y = \frac{u}{2} + \frac{2}{u} \) if \( u = x - x^2 \)

d) \( y^5 - 3x^2 = x \)

2) Find two positive numbers, \( x \) and \( y \), whose sum is 100 and whose product is as large as possible. (10 points)

3) Given the cost function \( C(x) = x^3 - 6x^2 + 13x + 15 \), find the minimum marginal cost. (10 points)

4) The monthly advertising revenue \( A \) and the monthly circulation \( x \) of a magazine are related approximately by the equation \( A = 6\sqrt{x^2 - 400} \), \( x \geq 20 \), where \( A \) is given in thousands of dollars and \( x \) is measured in thousands of copies sold. At what rate is the advertising revenue changing if the current circulation is \( x = 25 \) thousand copies and the circulation is growing at the rate of 2 thousand copies per month? (10 points)

5) The length, \( x \), of the edge of a cube is increasing. For what value of \( x \) is \( \frac{dV}{dt} \) equal to 12 times the rate of increase of \( x \)? (Note: Volume\_cube = length\(^3\)) (10 points)

6) Sketch the curve \( y = x^4 - 6x^2 \). Be sure to include all relevant details on the graph including at least the following: relative extrema, inflection points, concavity, and \( y \)-intercept. (10 points)

7) A California distributor of sporting equipment expects to sell 10,000 cases of tennis balls during the coming year at a steady rate. Yearly carry costs (to be computed on the average number of cases in stock during the year) are $10 per case, and the cost of placing an order with the manufacturer is $80. Determine the economic order quantity (EOQ), that is, the order quantity that minimizes the inventory costs and then find the minimum inventory cost. (10 points)
Extra Credit: (2 points each)

1) Find the quadratic function \( f(x) = ax^2 + bx + c \) that goes through \((2, 0)\) and has a local maximum at \((0, 1)\).

2) Let \( f(x), g(x), \) and \( h(x) \) be differentiable functions. Find a formula for the derivative of \( f(x)g(x)h(x) \).

3) A sugar refinery can produce \( x \) tons of sugar per week at a weekly cost of \( 0.1x^2 + 5x + 2250 \) dollars. Find the level of production for which the average cost is at a minimum.

**SLO Assessment Results for Test 3:**

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the test</th>
<th>Students who 'knew how to do this'</th>
<th>Students who 'had some idea how to do this but need more practice'</th>
<th>Students who 'did not know how to do this'</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph a function – indicate the relative extrema, inflection points, concavity, and y-intercept (MPO 1.1): question 6</td>
<td>7 = 23%</td>
<td>17 = 57%</td>
<td>6 = 20%</td>
</tr>
<tr>
<td>Calculate derivatives using the product rule (MPO 1.5): question 1a &amp; EC2</td>
<td>24 = 80%</td>
<td>3 = 10%</td>
<td>3 = 10%</td>
</tr>
<tr>
<td>Calculate derivatives using the quotient rule (MPO 1.5): question 1b</td>
<td>24 = 80%</td>
<td>2 = 7%</td>
<td>4 = 13%</td>
</tr>
<tr>
<td>Calculate derivatives using the chain rule (MPO 1.5): question 1c</td>
<td>10 = 48%</td>
<td>5 = 24%</td>
<td>6 = 29%</td>
</tr>
<tr>
<td>Calculate derivatives using implicit differentiation (MPO 1.5): question 1d</td>
<td>18 = 62%</td>
<td>4 = 14%</td>
<td>7 = 24%</td>
</tr>
<tr>
<td>Solve related rates problems (MPO 2.3) questions 4 &amp; 5</td>
<td>1 = 3%</td>
<td>13 = 43%</td>
<td>16 = 53%</td>
</tr>
<tr>
<td>Solve optimization problems (MPO 2.4): questions 2, 3 &amp; EC3</td>
<td>17 = 57%</td>
<td>9 = 30%</td>
<td>4 = 13%</td>
</tr>
<tr>
<td>Solve economic order quantity problems (EOQ) (MPO 2.4) question 7</td>
<td>20 = 67%</td>
<td>4 = 13%</td>
<td>6 = 20%</td>
</tr>
</tbody>
</table>

**NOTE:** Question EC1 was not blueprinted on this test.
1) Solve each equation below for $x$. Please be sure to give exact (not rounded) values. (6 points each)

   a) $3(2.7)^{5x} = 8.1$

   b) $e^{2x} = 5$

   c) $\ln(4-x) = \frac{1}{2}$

   d) $\ln(x+1) - \ln(x-2) = 1$

2) Find $\frac{dy}{dx}$ given the following: (6 points each)

   a) $y = \frac{e^{4x}}{4+x}$

   b) $y = e^x (1+x)^2$

   c) $y = \ln(e^{5x} + 1)$

   d) $y = \ln \left( \frac{x-1}{x+1} \right)$

   e) $y = \ln \left( \frac{x+1}{e^{x+1}} \right)$

   f) $y = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}}$ (NOTE: You may use logarithmic differentiation.)

3) Find the slope of the tangent line to the curve $y = xe^x$ at $(0,0)$. (10 points)

4) Find the coordinates of the minimum point of $y = -5x + e^x$. (10 points)
5) If the demand equation for a certain commodity is \( p = \frac{45}{\ln x} \), determine the marginal revenue function for this commodity, and compute the marginal revenue when \( x = 20 \). (10 points)

6) A painting purchased in 1998 for $100,000 is estimated to be worth \( v(t) = 100,000e^{\frac{t}{5}} \) dollars after \( t \) years. At what rate will the painting be appreciating in 2008? (10 points)

Extra Credit: (3 points each)

1) Find the point on the graph of \( y = e^{-x} \) where the tangent line has slope – 2.

2) Find values of \( h \) and \( k \) for which the graph of \( y = he^{kx} \) passes through the points \((1,6)\) and \((4,48)\).

### SLO Assessment Results for Test 4:

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the test</th>
<th>Students who ‘knew how to do this’</th>
<th>Students who ‘had some idea how to do this but need more practice’</th>
<th>Students who ‘did not know how to do this’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve exponential equations (MPO 1.2): questions 1a &amp; 1b</td>
<td>15 = 54%</td>
<td>11 = 39%</td>
<td>2 = 7%</td>
</tr>
<tr>
<td>Solve logarithmic equations (MPO 1.2): questions 1c &amp; 1d</td>
<td>10 = 36%</td>
<td>12 = 43%</td>
<td>6 = 21%</td>
</tr>
<tr>
<td>Calculate derivatives of exponential functions (MPO 1.5): questions 2a, 2b &amp; 4</td>
<td>19 = 68%</td>
<td>7 = 25%</td>
<td>2 = 7%</td>
</tr>
<tr>
<td>Calculate derivatives of logarithmic functions (MPO 1.5): questions 2c, 2d, 2e &amp; 5</td>
<td>5 = 18%</td>
<td>14 = 50%</td>
<td>9 = 32%</td>
</tr>
<tr>
<td>Calculate derivatives using logarithmic differentiation (MPO 1.5): question 2f</td>
<td>12 = 43%</td>
<td>7 = 25%</td>
<td>9 = 32%</td>
</tr>
<tr>
<td>Calculate derivatives using the product rule (MPO 1.5): questions 2b &amp; 3</td>
<td>17 = 61%</td>
<td>9 = 32%</td>
<td>2 = 7%</td>
</tr>
<tr>
<td>Calculate derivatives using the quotient rule (MPO 1.5): question 2a</td>
<td>17 = 61%</td>
<td>9 = 32%</td>
<td>2 = 7%</td>
</tr>
<tr>
<td>Calculate derivatives using the chain rule (MPO 1.5): questions 2a, 2c &amp; 6</td>
<td>22 = 79%</td>
<td>4 = 14%</td>
<td>2 = 7%</td>
</tr>
</tbody>
</table>
Name: ____________________________

Take-Home Test on Sections 5.1 – 5.3 & 6.1 – 6.3 Math 127 Dr. Gaulden

PLEASE show all work to receive full credit. That means that if you do not show all work, you will not receive full credit. Work carefully and neatly. Good luck!

1) Determine each integral given below. (5 points each)
   a) \[ \int \left( 4x^3 + \frac{1}{2}\sqrt{x} - 3 \right) dx \]
   b) \[ \int \left( 2x - \frac{1}{x} \right) dx \]

2) Find all functions \( f(x) \) such that \( f''(x) = 3x^2 + e^{-x} \) and \( f(0) = 2 \). (10 points)

3) Find the exact area (do not approximate with a Riemann sum and do not use a calculator and round your answer) under the curve \( y = x^2 \) from \( x = 0 \) to \( x = 5 \). (10 points)

4) Use a Riemann sum to approximate the area under the graph of \( y = x^3 \) in the interval \( 0 \leq x \leq 2 \) with \( n = 4 \) (that is, 4 subintervals or “panels”) and use midpoints of subintervals. (10 points)

5) How much money must you invest now at 6.2% interest compounded continuously to have $10,000 at the end of 8 years? (10 points)

6) An investment grows at a continuous 10% rate per year. In how many years will the value of the investment double? (10 points)

7) A movie theater has a seating capacity of 3,000 people. The number of people attending a show at price \( p \) dollars per ticket is \( q = \frac{16,000}{p} - 1200 \). Currently the price is $8 per ticket. If the price is lowered, will revenue increase or decrease? (10 points)

8) Suppose that the marginal cost function of a computer manufacturer is \( C'(x) = \frac{4}{5}x^2 - x + 150 \) dollars per unit at production level \( x \) (where \( x \) is measured in units of 100 computers). Determine the total cost of producing 10 additional units if 10 units are currently being produced. (10 points)

9) Let \( P(t) \) be the population (in millions) of a certain city \( t \) years after 1992, and suppose that \( P(t) \) satisfies the differential equation \( P'(t) = 0.03P(t) \), \( P(0) = 4 \). Approximately what was the population in 2000? (10 points)
10) A ball is thrown upward from a height of 192 feet above the ground, with an initial velocity of 64 feet per second. From physics it is known that the velocity at time $t$ is $v(t) = 64 - 32t$ feet per second. How high will the ball go? (Hint: You must find $s(t)$, the function giving the height of the ball at time $t$, first to answer this question.) (10 points)

SLO Assessment Results for Test 5:

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the test</th>
<th>Students who ‘knew how to do this’</th>
<th>Students who ‘had some idea how to do this but need more practice’</th>
<th>Students who ‘did not know how to do this’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate Riemann sums to estimate definite integrals (MPO 1.6): question 4</td>
<td>19 = 83%</td>
<td>4 = 17%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Integrate functions (MPO 1.7): questions 1a &amp; 2</td>
<td>22 = 96%</td>
<td>1 = 4%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Calculate definite integrals by applying the Fundamental Theorem of Calculus (MPO 1.7): questions 1b, 3 &amp; 8</td>
<td>21 = 91%</td>
<td>2 = 9%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve compound interest problems (MPO 2.1): question 6</td>
<td>21 = 91%</td>
<td>2 = 9%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve present value problems (MPO 2.1): question 5</td>
<td>20 = 87%</td>
<td>3 = 13%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve growth and decay problems (MPO 2.5): question 9</td>
<td>23 = 100%</td>
<td>0 = 0%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve elasticity of demand problems (MPO 2.6): question 7</td>
<td>21 = 91%</td>
<td>1 = 4%</td>
<td>1 = 4%</td>
</tr>
</tbody>
</table>
1) Let \( f(x, y) = x^2 - 3xy - y^2 \). Compute \( f(5, -2) \) \( (10 \text{ points}) \)

2) The value of residential property for tax purposes is usually much lower than its actual market value. If \( v \) is the market value, the assessed value for real estate taxes might be only 40% of \( v \). Suppose that the property tax, \( T \), in a community is given by the function \( T = f(r, v, x) = \frac{r}{100} (0.40v - x) \), where \( v \) is the estimated market value of a property (in dollars), \( x \) is a homeowner’s exemption (a number of dollars depending on the type of property), and \( r \) is the tax rate (stated in dollars per hundred dollars) of net assessed value. Determine the real estate tax on a property valued at $200,000 with a homeowner’s exemption of $5000, assuming a tax rate of $3.00 per hundred dollars of net assessed value. \( (10 \text{ points}) \)

3) Set up the integral \( \text{(DO NOT SOLVE!)} \) used to find the area of the region between the curves \( y = 8x^2 \) and \( y = \sqrt{x} \). \( (10 \text{ points}) \)

4) Determine the area of the region bounded by the curves \( y = x^2 \) and \( y = x \). \( (12 \text{ points}) \)

5) Draw the level curve of the function \( f(x, y) = x - y \) containing the point \( (0, 0) \). \( (10 \text{ points}) \)

6) Find \( \frac{\partial f}{\partial x} \) and \( \frac{\partial f}{\partial z} \) if \( f(x, y, z) = x \ z \ e^z \). \( (12 \text{ points}) \)

7) Find all point(s) \( (x, y) \) where \( f(x, y) = 3x^2 - 6xy + y^3 - 9y \) has a possible relative maximum or minimum. Then use the second-derivative test to determine, if possible, the nature of \( f(x, y) \) at the point(s). If the second-derivative test is inconclusive, so state. \( (12 \text{ points}) \)
8) **CHOOSE** and complete **ONE** of the problems below. **NOTE:** If you complete more than one, only the first will be graded unless otherwise indicated. (12 points)

a) An investment pays 10% interest compounded continuously. If money is invested steadily at the rate of $5000 per year, how much time is required until the value of the investment reaches $140,000?

**OR**

b) Find the volume of the solid of revolution generated by revolving about the \( x \)-axis the region under the curve \( y = e^{-x} \) from \( x = 0 \) to \( x = 1 \).

9) **CHOOSE** and complete **ONE** of the problems below. **NOTE:** If you complete more than one, only the first will be graded unless otherwise indicated. (12 points)

a) A company manufactures and sells two products, I and II, that sell for $10 and $9 per unit, respectively. The cost of producing \( x \) units of product I and \( y \) units of product II is \( 400 + 2x + 3y + 0.01(3x^2 + xy + 3y^2) \). Find the values of \( x \) and \( y \) that maximize the company's profits. (RECALL: Profit = Revenue – Cost)

**OR**

b) A farmer can produce \( f(x, y) = 200\sqrt{6x^2 + y^2} \) units of produce by utilizing \( x \) units of labor and \( y \) units of capital. (The capital is used to rent or purchase land, materials, and equipment.) Calculate the marginal productivities of labor and capital when \( x = 10 \) and \( y = 5 \).

**Extra Credit:**

1) Find a formula \( C(x, y, z) \) that gives the cost of materials for a **closed** rectangular box with length \( x \) feet, width \( y \) feet, and height \( z \) feet. Assume that the material for the top and bottom costs $3 per square foot and the material for the sides costs $5 per square foot. (3 points)

2) Minimize \( x^2 + 3y^2 + 10 \) subject to the constraint \( 8 - x - y = 0 \). (NOTE: The Lagrange multiplier method may be used to solve this problem.) (3 points)
### SLO Assessment Results for Test 6:

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the test</th>
<th>Students who ‘knew how to do this’</th>
<th>Students who ‘had some idea how to do this but need more practice’</th>
<th>Students who ‘did not know how to do this’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluate multi-variable functions (MPO 1.1): questions 1 &amp; 2</td>
<td>14 = 70%</td>
<td>6 = 30%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Determine relative minima and relative maxima of multi-variable functions (MPO 1.1): question 7</td>
<td>4 = 20%</td>
<td>12 = 60%</td>
<td>4 = 20%</td>
</tr>
<tr>
<td>Calculate partial derivatives (MPO 1.5): questions 6 &amp; 9b</td>
<td>6 = 30%</td>
<td>13 = 65%</td>
<td>1 = 5%</td>
</tr>
<tr>
<td>Determine the areas between curves (MPO 1.7): questions 3 &amp; 4</td>
<td>7 = 35%</td>
<td>13 = 65%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Determine a specified volume of revolution (MPO 1.8): question 8b</td>
<td>5 = 63%</td>
<td>2 = 25%</td>
<td>1 = 13%</td>
</tr>
<tr>
<td>Solve continuous income stream future value problems (MPO 2.1): question 8a</td>
<td>3 = 25%</td>
<td>6 = 50%</td>
<td>3 = 25%</td>
</tr>
<tr>
<td>Solve optimization problems (MPO 2.4) questions 9a &amp; EC2</td>
<td>7 = 54%</td>
<td>2 = 15%</td>
<td>4 = 31%</td>
</tr>
</tbody>
</table>

**NOTE:** Questions 5 & EC1 were not blueprinted on this test.
1) Differentiate as indicated below. (6 points each)

a) Given \( y = (2x + 4)^3 + \frac{x-1}{x+1} + x\sqrt{x+1} \), determine \( \frac{dy}{dx} \).

b) Given \( y = \frac{(x+1)(2x+1)(3x+1)}{\sqrt{4x+1}} \), use logarithmic differentiation to determine \( \frac{dy}{dx} \).

c) Given \( f(x,y,z) = xy^2z \), evaluate \( \frac{\partial f}{\partial y} \) at \((x, y, z) = (2, -1, 3)\).

2) Integrate as indicated below. (6 points each)

a) \( \int_{1}^{3} \left( x\sqrt{x} - 4x^3 + \frac{1}{3x} + \frac{x}{2} \right) \, dx \)

b) \( \int_{0}^{1} 4e^{-3x} \, dx \) \( \text{NOTE: Do not round your answer – give the exact value.} \)

3) Determine the function values/continuity as indicated below. (8 points each)

a) Let \( g(x,y,z) = \frac{x}{y-z} \). Compute \( g(2,3,4) \).

b) Given \( f(x) = \begin{cases} 2x-1 & \text{for } 0 \leq x \leq 1 \\ 1 & \text{for } 1 < x \end{cases} \), determine whether \( f(x) \) is continuous at \( x = 1 \). Be sure to show adequate work to justify your answer and receive full credit.

4) Solve each equation below. (8 points each)

a) \( \ln(4-x) = \frac{1}{2} \)

b) \( e^{x^2-2x} = e^8 \)
5) A movie theater has a seating capacity of 3000 people. The number of people attending a show at price \( p \) dollars per ticket is \( q = \frac{18000}{p} - 1500 \). Currently, the price is $6 per ticket. If the price is lowered, will revenue increase or decrease? Be sure to show adequate work to justify your answer and receive full credit. (9 points)

6) Suppose that the price \( p \) (in dollars) and the weekly sales \( x \) (in thousands of units) of a certain commodity satisfy the demand equation \( 2p^3 + x^2 = 4500 \). Determine the rate at which sales are changing at a time when \( x = 50, \ p = 10 \), and the price is falling at the rate of $0.50 per week. (9 points)

7) Choose and complete ONE of the two problems below. (Note: If you complete both problems, only the first one will be graded.) (10 points)

   a) Suppose that the marginal cost function of a handbag manufacturer is \( C'(x) = \frac{3}{32}x^2 - x + 200 \) dollars per unit at production level \( x \) (where \( x \) is measured in units of 100 handbags). Find the total cost of producing 6 additional units if 2 units are currently being produced.

   OR

   b) Let \( C(x) = 12x + 1100 \) denote the total cost (in dollars) of manufacturing \( x \) units of a certain commodity per day. What is the total cost if the production is set at 10 units per day? Also, what is the marginal cost?

8) Choose and complete ONE of the two problems below. (Note: If you complete both problems, only the first one will be graded.) (10 points)

   a) Shakespear’s Pizza sells 1000 large vegi pizzas per week for $18 a pizza. When the owner offers a $5 discount, the weekly sales increase to 1500. Assume a linear relation between the weekly sales \( A(x) \) and the discount \( x \). Find the value of \( x \) that maximizes the weekly revenue. OR

   b) $1000 is deposited in a savings account at 6% interest compounded continuously. How many years are required for the balance in the account to reach $2500?

Extra Credit: (3 points each)

1) Use limits (the definition of the derivative) to compute \( f''(3) \) where \( f(x) = x^2 + 1 \). Note: To receive credit for this problem, limits (the definition of the derivative) and not the power rule must be used to compute \( f''(3) \).

2) 100 shares of a technology stock were purchased on January 2, 1990, for $1200 and sold on January 2, 1998, for $12,500. What rate of interest compounded continuously did this investment earn?
### SLO Assessment Results for the Final Exam:

<table>
<thead>
<tr>
<th>Skill/Math Topic (part of specified MPO): blueprinted questions from the final exam</th>
<th>Students who ‘knew how to do this’</th>
<th>Students who ‘had some idea how to do this but need more practice’</th>
<th>Students who ‘did not know how to do this’</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evaluate multi-variable functions (MPO 1.1) question 3a</td>
<td>20 = 100%</td>
<td>0 = 0%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve quadratic equations (MPO 1.2) question 4b</td>
<td>15 = 75%</td>
<td>0 = 0%</td>
<td>5 = 25%</td>
</tr>
<tr>
<td>Solve exponential equations (MPO 1.2) question 4b</td>
<td>20 = 100%</td>
<td>0 = 0%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve logarithmic equations (MPO 1.2) question 4a</td>
<td>14 = 70%</td>
<td>3 = 15%</td>
<td>3 = 15%</td>
</tr>
<tr>
<td>Determine whether a function is continuous at a specified value of x (MPO 1.3) question 3b</td>
<td>7 = 35%</td>
<td>5 = 25%</td>
<td>8 = 40%</td>
</tr>
<tr>
<td>Determine the derivative of a function by using limits and difference quotients (MPO 1.4) question EC1</td>
<td>0 = 0%</td>
<td>0 = 0%</td>
<td>9 = 100%</td>
</tr>
<tr>
<td>Calculate derivatives using the product rule (MPO 1.5): question 1a</td>
<td>7 = 35%</td>
<td>5 = 25%</td>
<td>8 = 40%</td>
</tr>
<tr>
<td>Calculate derivatives using the quotient rule (MPO 1.5): question 1a</td>
<td>9 = 45%</td>
<td>5 = 25%</td>
<td>6 = 30%</td>
</tr>
<tr>
<td>Calculate derivatives using the chain rule (MPO 1.5): question 1a</td>
<td>13 = 65%</td>
<td>5 = 25%</td>
<td>2 = 10%</td>
</tr>
<tr>
<td>Calculate derivatives using logarithmic differentiation (MPO 1.5): question 1b</td>
<td>8 = 40%</td>
<td>6 = 30%</td>
<td>6 = 30%</td>
</tr>
<tr>
<td>Calculate partial derivatives (MPO 1.5): question 1c</td>
<td>11 = 55%</td>
<td>4 = 20%</td>
<td>5 = 25%</td>
</tr>
<tr>
<td>Integrate functions (MPO 1.7): question 2a</td>
<td>3 = 15%</td>
<td>10 = 50%</td>
<td>7 = 35%</td>
</tr>
<tr>
<td>Calculate definite integrals by applying the Fundamental Theorem of Calculus (MPO 1.7) question 2b</td>
<td>11 = 55%</td>
<td>5 = 25%</td>
<td>4 = 20%</td>
</tr>
<tr>
<td>Solve compound interest problems (MPO 2.1) questions 8b &amp; EC2</td>
<td>10 = 50%</td>
<td>4 = 20%</td>
<td>6 = 30%</td>
</tr>
<tr>
<td>Solve marginal cost problems (MPO 2.2) question 7a or 7b</td>
<td>14 = 70%</td>
<td>6 = 30%</td>
<td>0 = 0%</td>
</tr>
<tr>
<td>Solve related rates problems (MPO 2.3) question 6</td>
<td>7 = 35%</td>
<td>2 = 10%</td>
<td>11 = 55%</td>
</tr>
<tr>
<td>Skill/Math Topic (part of specified MPO): blueprinted questions from the final exam</td>
<td>Students who ‘knew how to do this’</td>
<td>Students who ‘had some idea how to do this but need more practice’</td>
<td>Students who ‘did not know how to do this’</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Solve optimization problems (MPO 2.4) question 8a</td>
<td>1 = 50%</td>
<td>0 = 0%</td>
<td>1 = 50%</td>
</tr>
<tr>
<td>Solve elasticity of demand problems (MPO 2.6) question 5</td>
<td>10 = 50%</td>
<td>4 = 20%</td>
<td>6 = 30%</td>
</tr>
</tbody>
</table>